Subject Code: NCE 202

Subject Name: Hydraulics & Hydraulic Machines

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UNIT -2

ENERGY DEPTH RELATIONSHIP

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INTRODUCTION

- WE have discussed Prismatic Channels in which bed slope and cross sectional shape do not change along channel length.
- Natural channels varies widely.
- But even man made channels may have to go transition due to engineering and economic considerations.
- For e.g. width of canal can be reduced when it crosses river or road, to reduce width of aqueduct or length of bridge.
- Reduction in width may be accompanied by raising or lowering of the bed to adjust the water level.

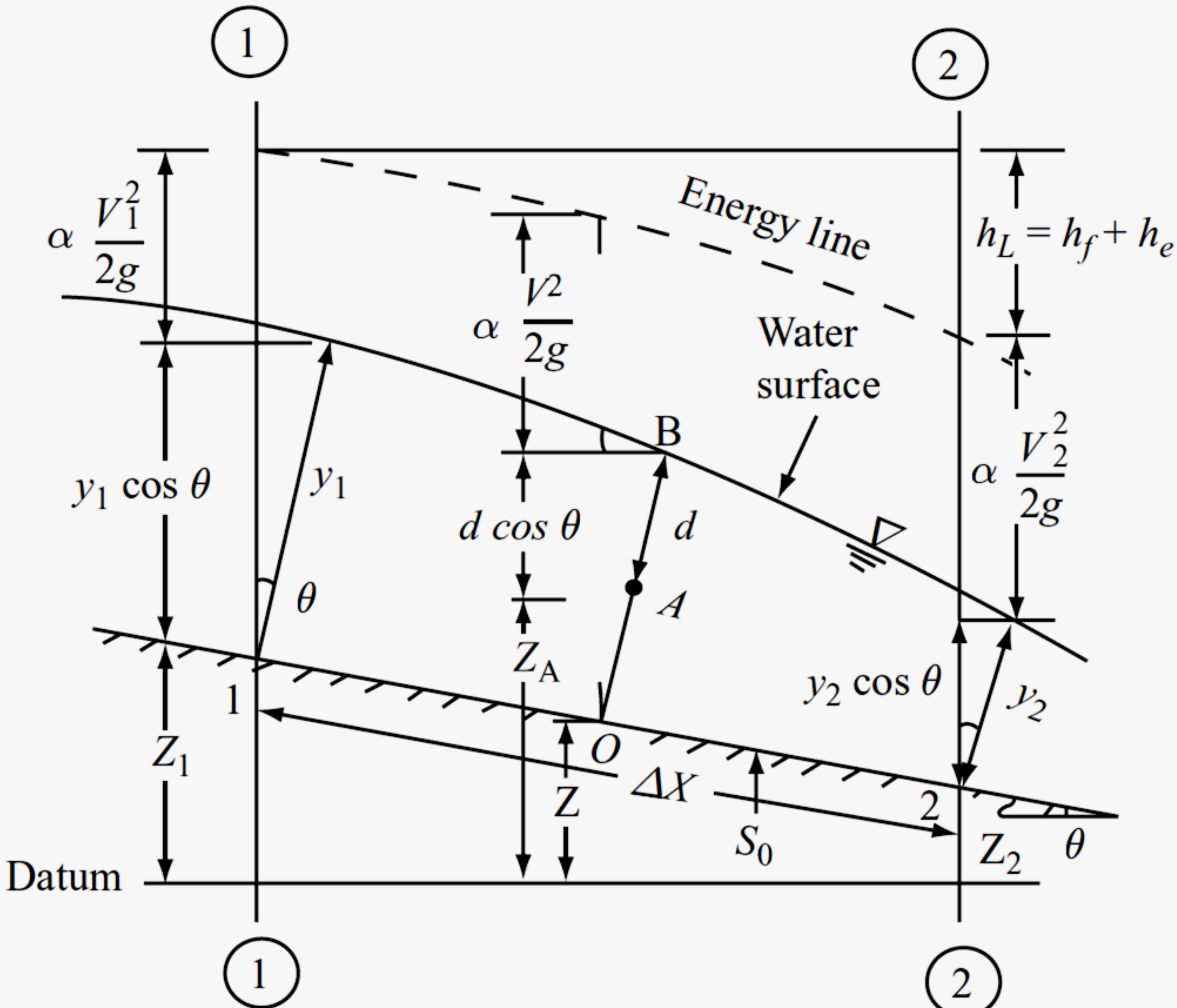


Fig.1.14 *Definition sketch for the energy equation*

SPECIFIC ENERGY

Total Energy

$$H = Z + y\cos\theta + \alpha\frac{V^2}{2g}$$

Assume datum coincides with channel bottom

$$E = y\cos\theta + \alpha\frac{V^2}{2g}$$

E is specific energy

When $\cos\theta = 1.0$ and $\alpha = 1.0$,

$$E = y + \frac{V^2}{2g}$$

Specific Energy in a channel is defined as the energy per kg of water at any section increased w.r.t. channel bottom, Z = 0

SPECIFIC ENERGY

In a channel with constant discharge, Q

$$Q = A_1 V_1 = A_2 V_2$$

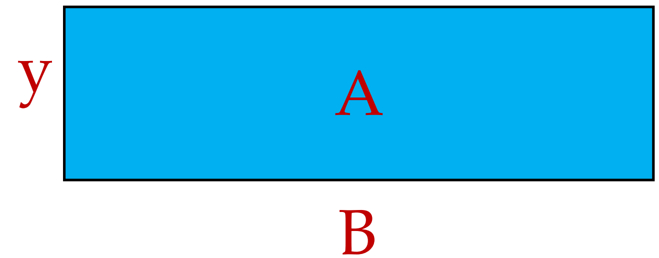
$$E = y + \frac{V^2}{2g} \longrightarrow E = y + \frac{Q^2}{2gA^2}$$

where A=f(y)

Consider rectangular channel (A=By) and Q=qB

q is the discharge per unit width of channel

$$E = y + \frac{q^2}{2gy^2}$$



3 roots (one is negative)

For a given Q and channel section E depends on y only

SPECIFIC ENERGY VS DEPTH

$$E = y + \frac{q^2}{2gy^2}$$

$$(E - y)\, y^2 = \frac{q^2}{2g} = constant$$

$$Ey^2 - y^3 = constant$$

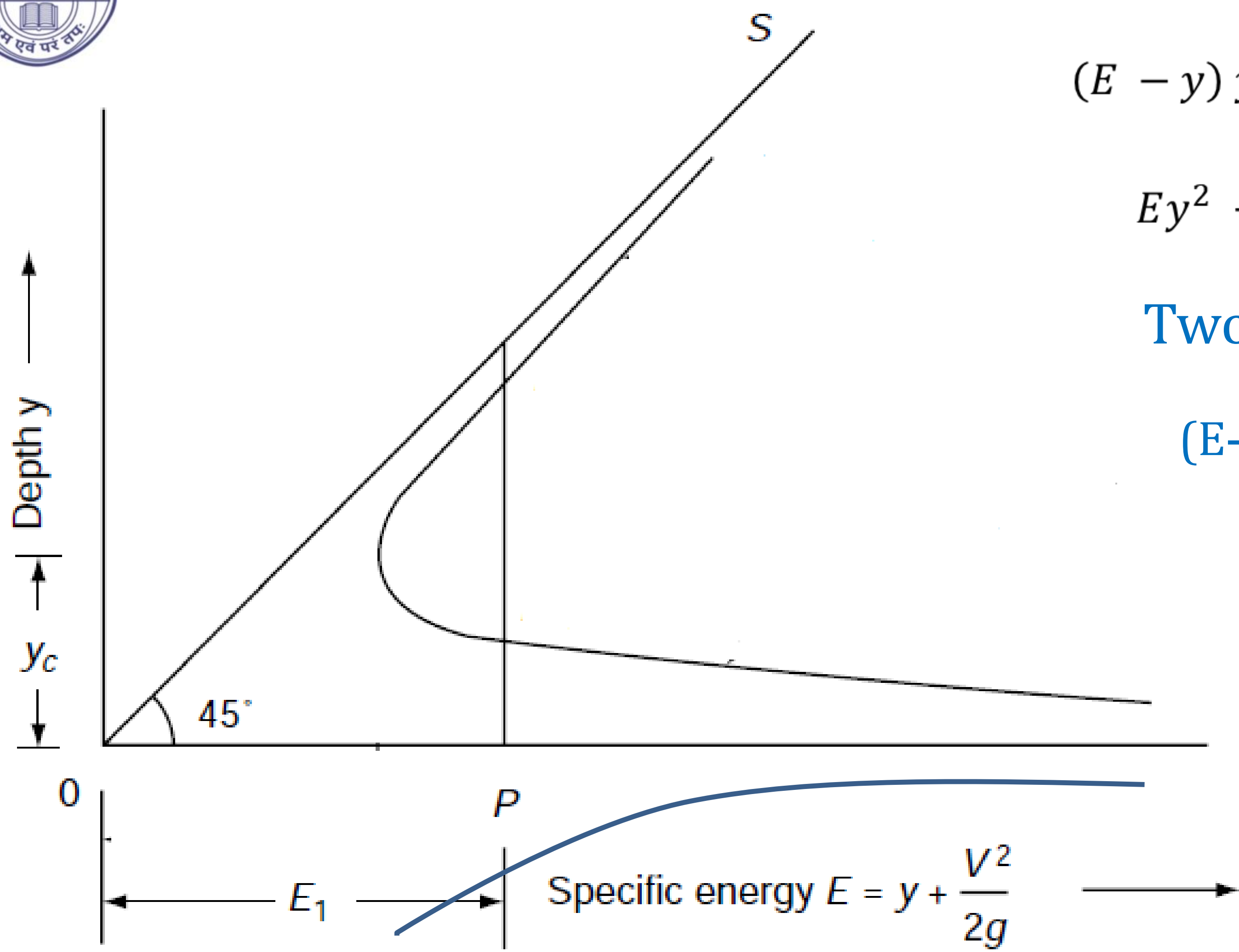


Fig. 2.1 *Definition sketch of specific energy*

Two asymptotes

(E-y) = 0 and y = 0

Because of assumption that θ is small the angle is 45^0 otherwise it will be different

SPECIFIC ENERGY VS DEPTH

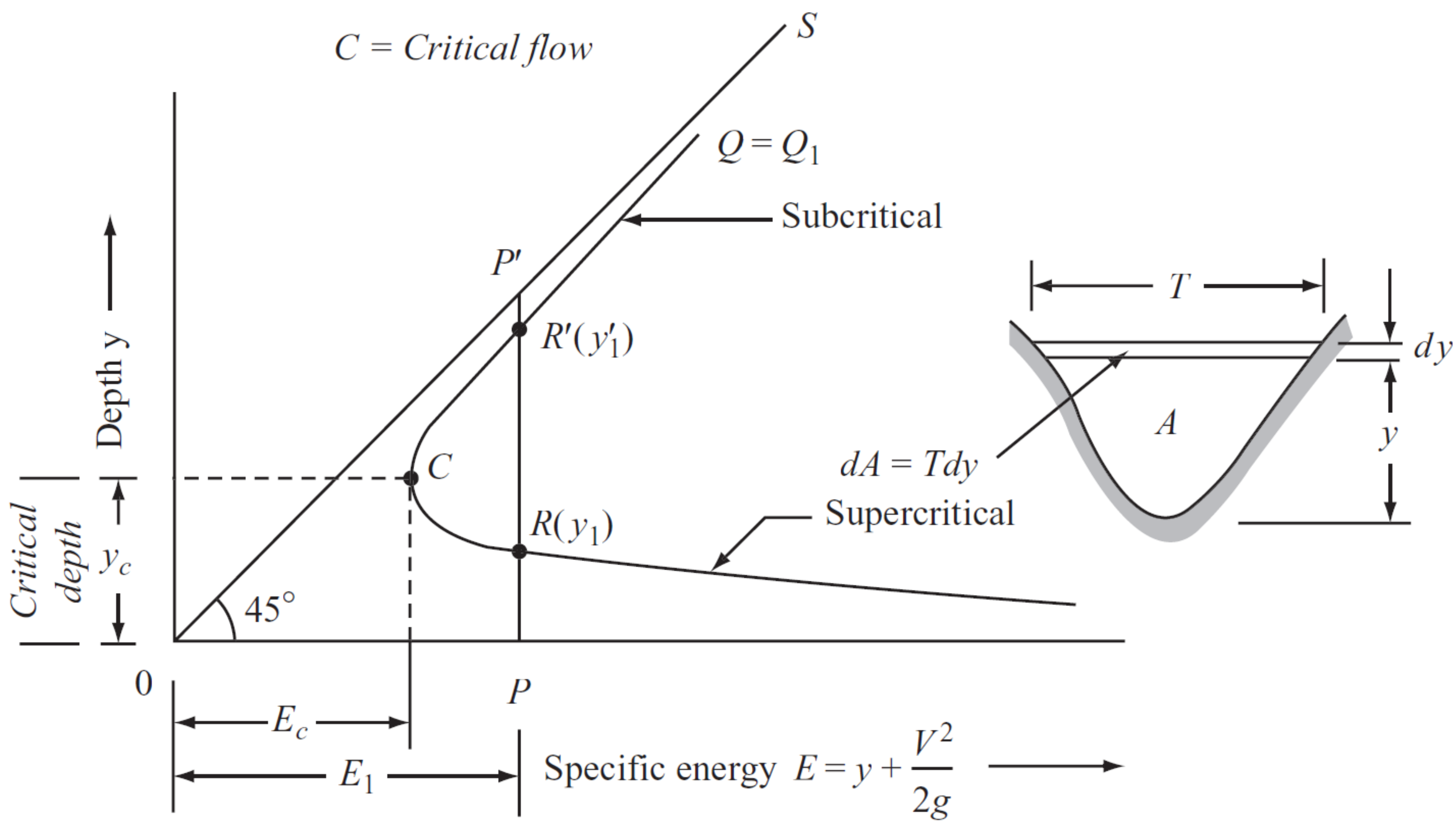


Fig. 2.1 *Definition sketch of specific energy*

SPECIFIC ENERGY VS DEPTH

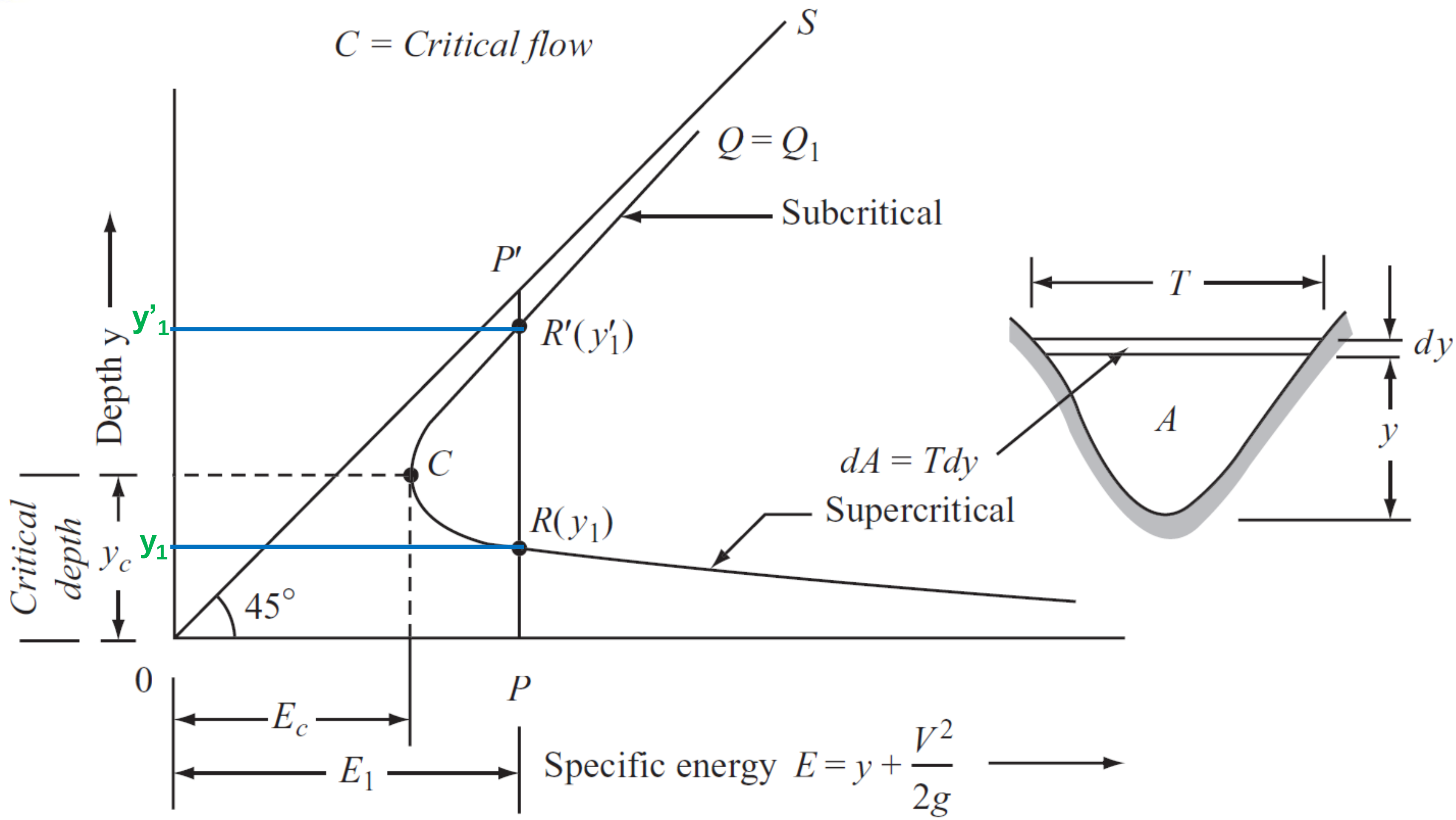


Fig. 2.1 *Definition sketch of specific energy*

SPECIFIC ENERGY VS DEPTH

- Two possible depths at same specific energy: Alternate Depths.
- At 45^0 line $E = y$
- P'R' or P'R are velocity heads
- No y for $E < E_c$
- E_c is minimum specific energy

SPECIFIC ENERGY vs DEPTH

For a rectangular channel

$$\frac{dE}{dy} = 0$$ For minimum Specific Energy

$$\Rightarrow 1 - \frac{q^2}{gy^3} \Rightarrow y_c = \left(\frac{q^2}{g}\right)^{1/3}$$ Critical depth of flow

$$\frac{V^2}{2g} = \frac{q^2}{2gy2_c} = \frac{y_c}{2}$$

So, $$E_c = \frac{3}{2}\, y_c$$ Go to the E vs y curve

SPECIFIC ENERGY vs DEPTH

For any other shape of channels

$$E = y + \frac{V^2}{2g} \qquad = y + \frac{Q^2}{2gA^2}$$

For minimum Specific Energy $\qquad \frac{dE}{dy} = 0$

$$\Rightarrow \quad 1 - \frac{Q^2}{gA^3} \cdot \frac{dA}{dy} = 0 \qquad \frac{dA}{dy} = T$$

So , $\boxed{\frac{Q^2 T}{gA^3} = 1}$ Condition for Critical Flow

SPECIFIC ENERGY vs DEPTH

For any other shape of channels

$$\frac{V^2}{2g} = \frac{Q^2}{A^2 2g} \qquad = \frac{gA}{T\,2g} \quad = \frac{A}{2T} \qquad = \frac{D}{2} \qquad D = \frac{A}{T}$$

D is Hydraulic Depth

So, $$E_c = y_c + \frac{D_c}{2}$$

$$E = y + \frac{Q^2}{2gA^2} \Longrightarrow Q = A\sqrt{2g(E-y)}$$

condition for maximum discharge differentiating with respect to y and equating it to zero while keeping $E =$ constant.

$$\frac{dQ}{dy} = \sqrt{2g(E-y)}\,\frac{dA}{dy} - \frac{gA}{\sqrt{2g(E-y)}} = 0$$

$$\frac{dA}{dy} = T \quad \text{and} \quad \frac{Q}{A} = \sqrt{2g(E-y)} \qquad \frac{Q^2 T}{gA^3} = 1.0$$

SPECIFIC ENERGY VS DEPTH

Froude number

$$Froude\ number = \sqrt{\frac{Inertia\ Force}{Gravitational\ Force}}$$

Its dimensionless number

Inertia Force = mass x acceleration = $\rho . AV . V = \rho . l^2 . V^2$ in form of dimension

Gravitational Force = mass x acceleration = $\rho . AV . g = \rho . l^3 . g$ in form of dimension

$$Froude\ number = \sqrt{\frac{V^2}{l.g}}$$

So, $Fr = \frac{V}{\sqrt{gy}}$ ➡ $Fr = \frac{V}{\sqrt{gD}}$ ➡ $Fr = \frac{V}{\sqrt{g\frac{A}{T}}}$

D is hydraulic Depth

SPECIFIC ENERGY VS DEPTH

More general definition of Fr $\dfrac{Q^2T}{gA^3} = Fr^2$

$\Rightarrow$ $\dfrac{V^2T}{gA} = Fr^2$ $\dfrac{A}{T} = D$ Hydraulic Depth

channel with large longitudinal slope θ and having a flow with an energy correction factor of α,

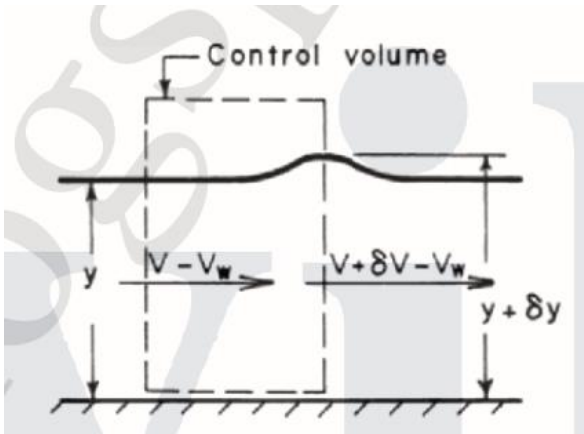
$$F = V \Big/ \left(\sqrt{\frac{1}{\alpha} g \frac{A}{T} \cos\theta} \right)$$

For critical flow: $1 = \dfrac{Q^2T}{gA^3} = Fr^2$ or $\dfrac{Q_c^2 T_c}{gA_c^3} = 1$

SPECIFIC ENERGY VS DEPTH

$y_1 < y_c$ ⟶ $V_1 > V_c$ ⟶ $F_r > 1$ supercritical region

$y'_1 > y_c$ ⟶ $V'_1 < V_c$ ⟶ $F_r < 1$ subcritical region

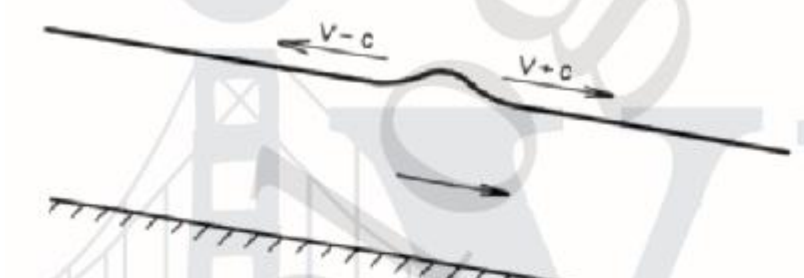


(b) Steady flow

$$V_w = V \pm \sqrt{gy}$$

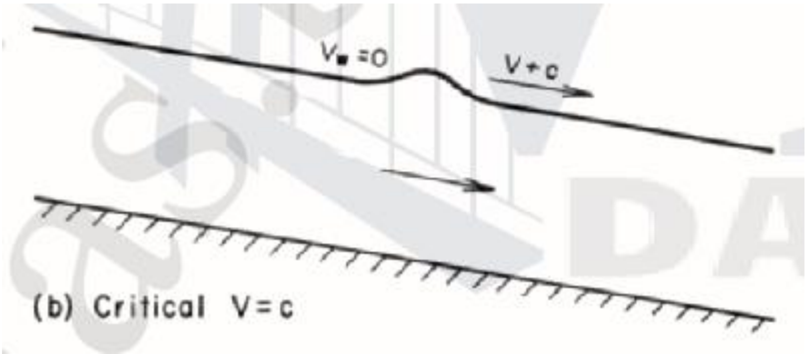
celerity, c, is the wave velocity relative to the medium in which the wave is traveling – i.e., $V_w = V \pm c$

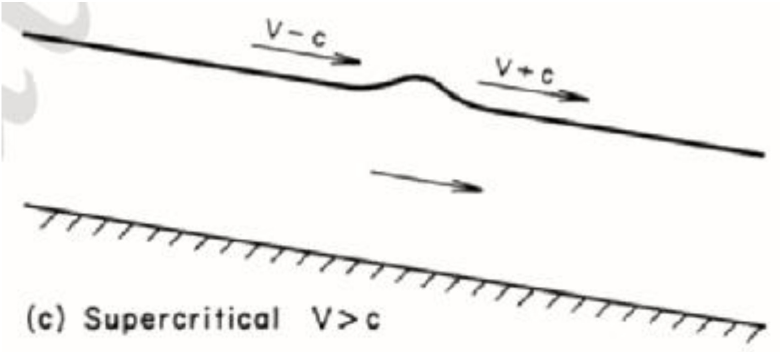
$$c = \sqrt{gy}$$



(a) Subcritical V < c

SPECIFIC ENERGY VS DEPTH





DISCHARGE as VARIABLE

$$E = y + \frac{Q^2}{2gA^2} \quad \Rightarrow \quad Q = A\sqrt{2g(E-y)}$$

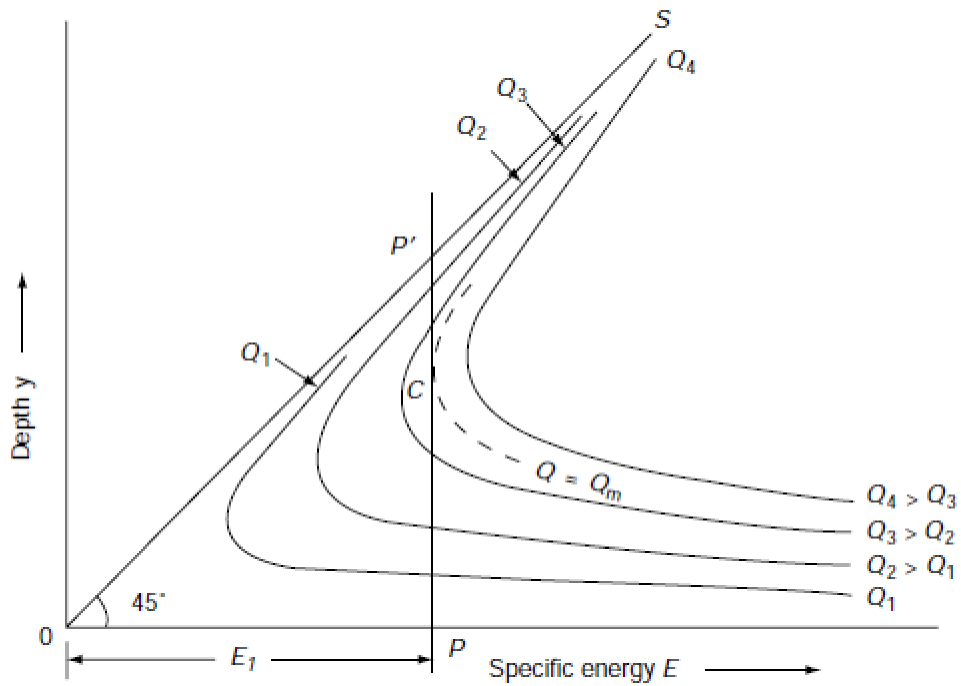


Fig. 2.2 *Specific energy for varying discharges*

EX. ALTERNATE DEPTH AND FROUDE NUMBER

Example 2.1 *A 2.5-m wide rectangular channel has a specific energy of 1.50 m when carrying a discharge of 6.48 m³/s. Calculate the alternate depths and corresponding Froude numbers.*

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gB^2y^2}$$

$$1.5 = y + \frac{(6.48)^2}{2 \times 9.81 \times (2.5)^2 y^2}$$

$$= y + \frac{0.34243}{y^2}$$

Solving this equation by trial and error, the alternate depths y_1 and y_2 are obtained as $y_1 = 1.296$ m and $y_2 = 0.625$ m.

Froude number $$F = \frac{V}{\sqrt{gy}} = \frac{6.48}{(2.5\,y)\sqrt{9.81\,y}} = \frac{0.82756}{y^{3/2}},$$

At $y_1 = 1.296$ m, $F_1 = 0.561$; and
at $y_2 = 0.625$ m, $F_2 = 1.675$

The depth $y_1 = 1.296$ m is in the subcritical flow region and the depth $y = 0.625$ m is in the supercritical flow region.

CALCULATION OF CRITICAL DEPTH

Rectangular Section

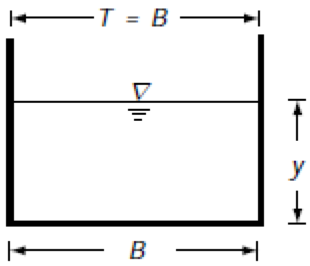


Fig. 2.3 *Rectangular channel*

$$\frac{Q^2 T_c}{g A_c^3} = \frac{V_c^2}{g y_c} = 1$$

$$\frac{V_c^2}{2g} = \frac{1}{2} y_c$$

Specific energy at critical depth $E_c = y_c + \frac{V_c^2}{2g} = \frac{3}{2} y_c$

, if q = discharge per unit width = Q/B,

$$\frac{q^2}{g} = y_c^3 \qquad y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

Since $A/T = y$, $\qquad F = \frac{V}{\sqrt{gy}}$

Triangular Channel

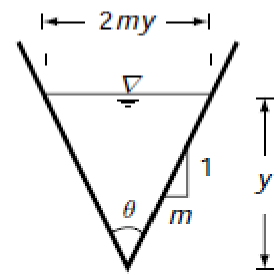


Fig. 2.4 *Triangular channel*

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c}$$

$$= \frac{m^3 y_c^6}{2 m y_c} \qquad = \frac{m^2 y_c^5}{2}$$

Hence $y_c = \left(\frac{2Q^2}{g m^2}\right)^{1/5}$

$$E_c = y_c + \frac{V_c^2}{2g}$$

$$= y_c + \frac{Q^2}{2 g A_c^2} \qquad = y_c + \frac{m^2 y_c^5}{4 m^2 y_c^4} \qquad E_c = 1.25 y_c$$

$$A/T = y/2, \qquad F = \frac{V\sqrt{2}}{\sqrt{gy}}$$

Circular Channel

Trapezoidal Channel

SECTION FACTOR Z

$$Z = A\sqrt{A/T}$$

At the critical-flow condition, $y = y_c$ and $Z_c = A_c\sqrt{A_c/T_c} = Q/\sqrt{g}$

FIRST HYDRAULIC EXPONENT M

$$Z^2 = C_1 y^M$$

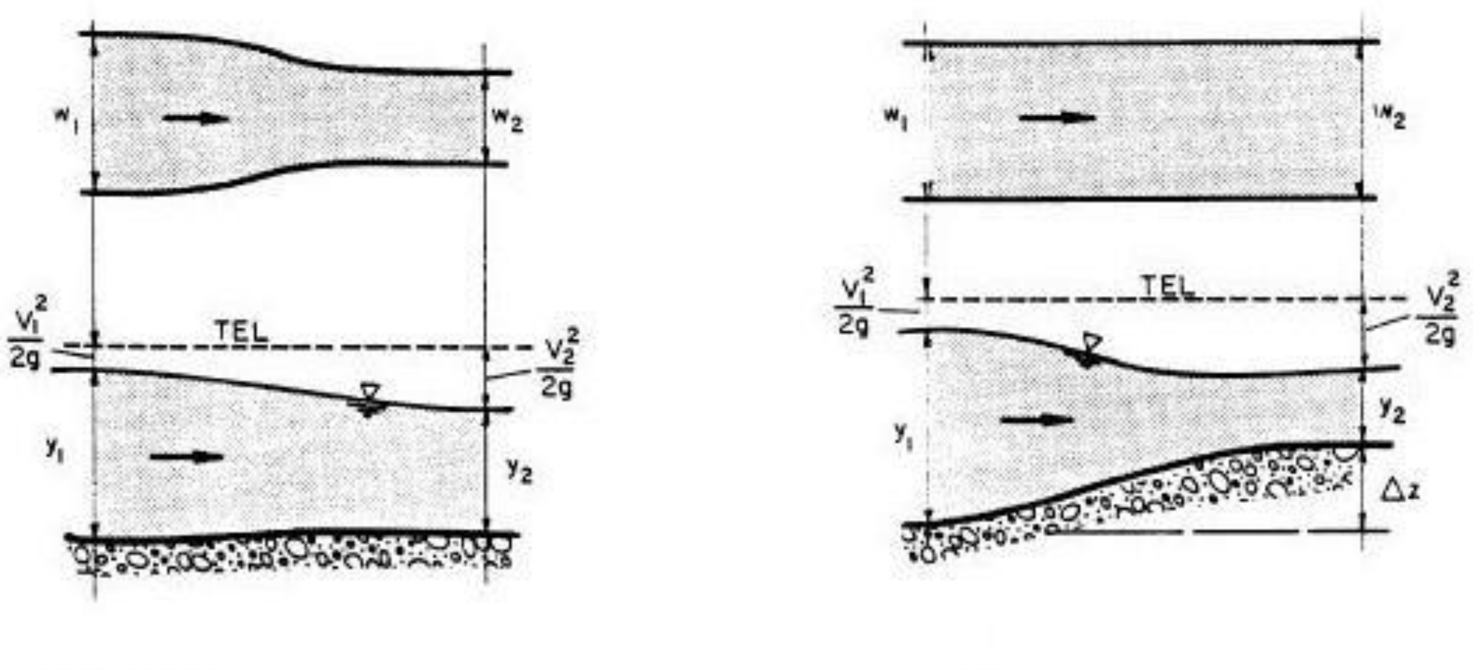
C_1 = a coefficient and M = an exponent called the *first hydraulic exponent.*

Example 2.3 *Obtain the value of the first hydraulic exponent M for (a) rectangular channel, and (b) an exponential channel where the area A is given as* $A = K_1 y^a$

$Z = A\sqrt{A/T}$ $\quad z^2 = \frac{A^3}{T} = B^2 y^3 = C_1 y^M$ $\quad M = 3.0$

TRANSITIONS

A transition is an open-channel flow structure whose purpose is to change the shape or cross-sectional area of the flow. The design objective is to avoid excessive energy losses and to minimize surface waves and other turbulence.



(a) Width contraction (b) Bed rise

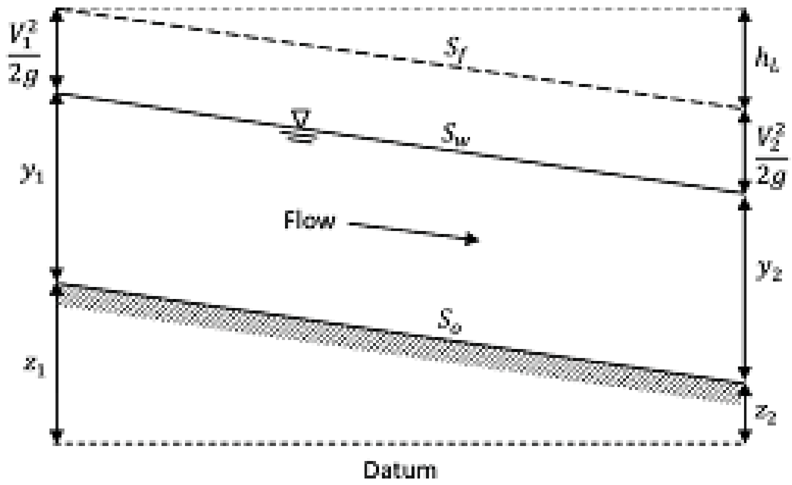
TRANSITIONS

CHANNEL WITH A HUMP – Subcritical Flow

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

If no loss of energy

$$E_1 = E_2$$



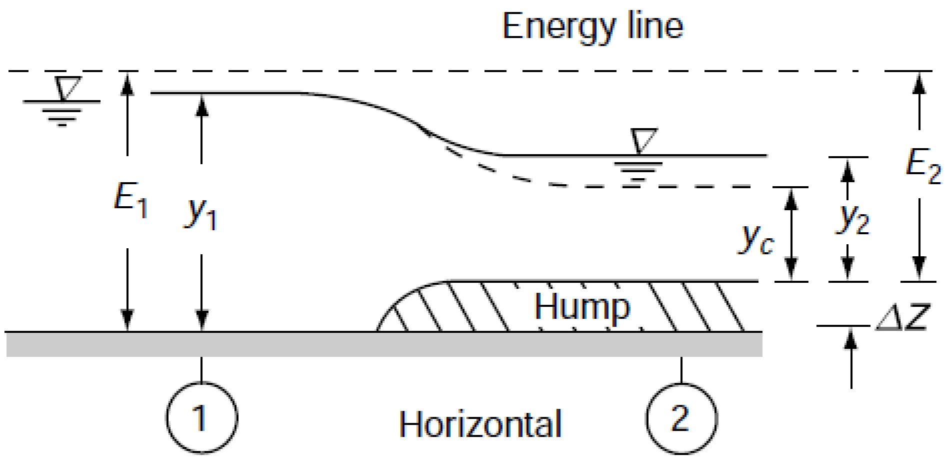


Fig. 2.9 *Channel transition with a hump*

TRANSITIONS

CHANNEL WITH A HUMP – Subcritical Flow

$$E_2 = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{Q^2}{2g\,B^2\,y_2^2}$$

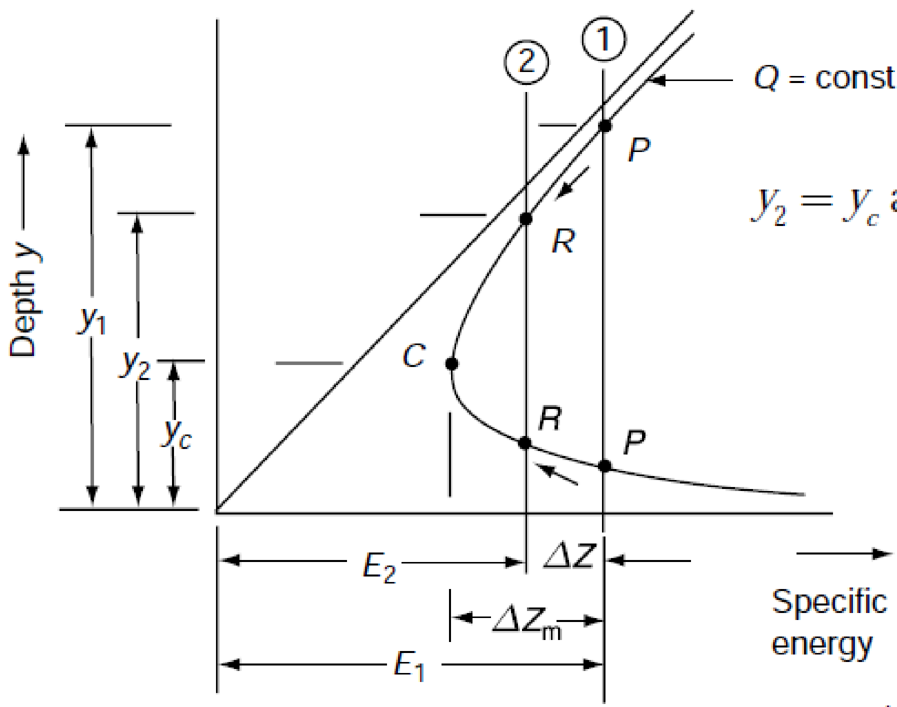


Fig. 2.10 *Specific-energy diagram for Fig. 2.9*

what happens when $\Delta Z > \Delta Z_m$

flow is not possible

$y_2 = y_c$ and

The upstream depth has to increase to cause an increase in the specific energy at Section 1.

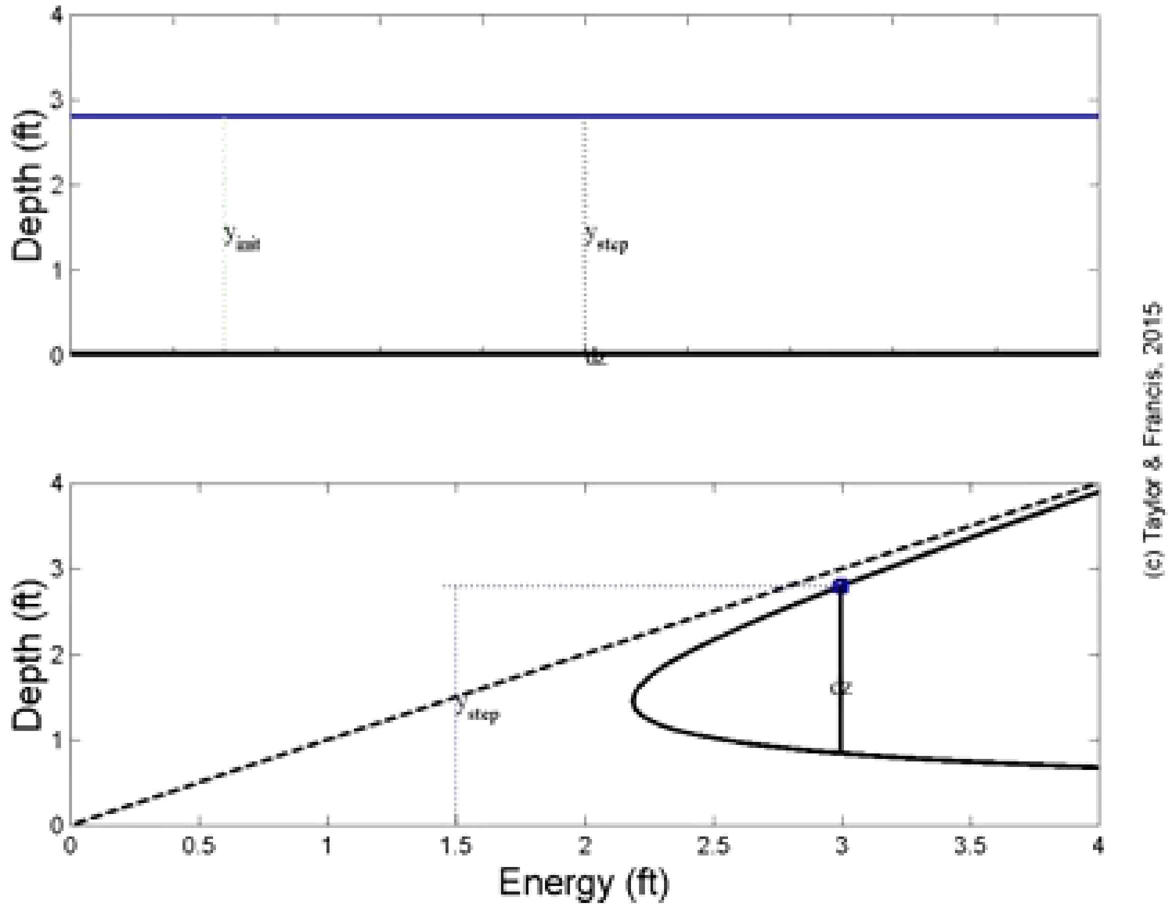
$$E_1' = y_1' + \frac{Q^2}{2gB^2\,y_1'^2}$$

$\{$with $E_1' > E_1$ and $y_1' > y_1\}$

At Section 2 the flow will continue at the minimum specific energy level, i.e. critical condition.

TRANSITIONS

CHANNEL WITH A HUMP – Subcritical Flow



TRANSITIONS

CHANNEL WITH A HUMP – Subcritical Flow

$$y_2 = y_c \text{ and } \quad E_1' - \Delta Z = E_2 = E_c = y_c + \frac{Q^2}{2g\,B^2\,y_c^2}$$

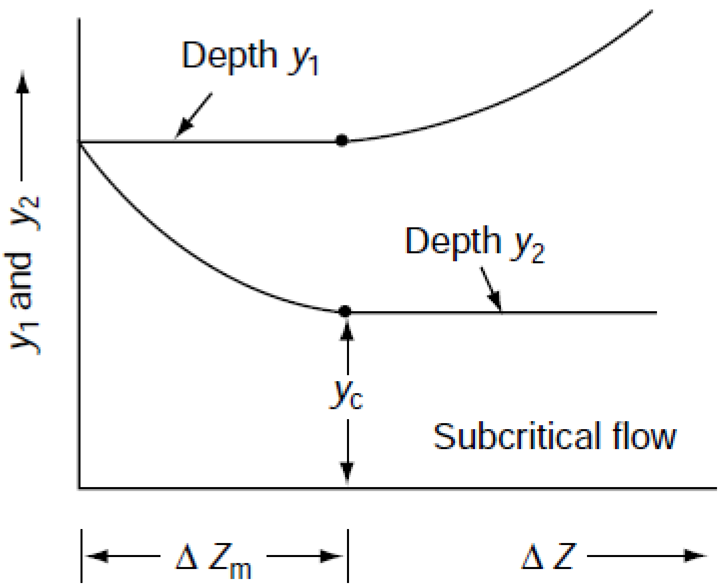


Fig. 2.11 *Variation of y_1 and y_2 in subcritical flow over a hump*

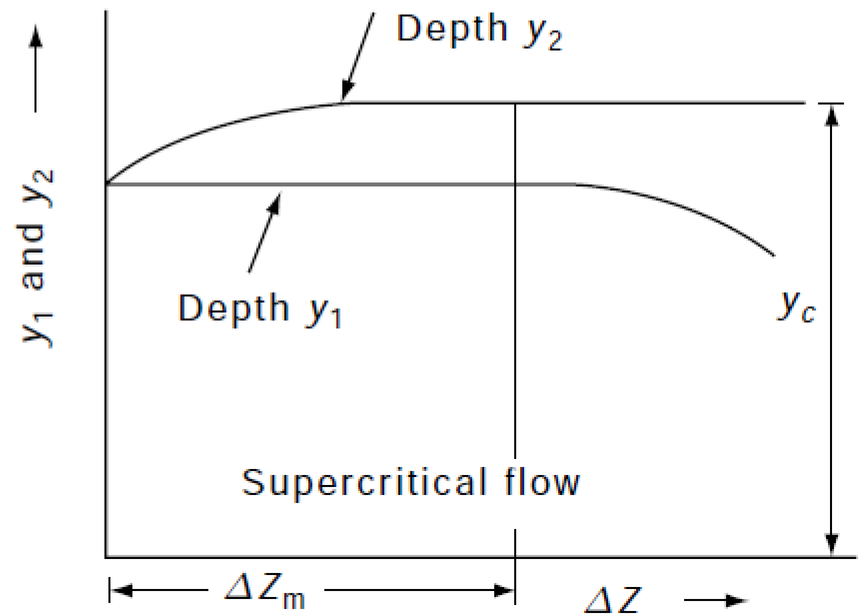


Fig. 2.12 *Variation of y_1 and y_2 in supercritical flow over a hump*

(b) Supercritical Flow

TRANSITIONS

Example 2.10 *A rectangular channel has a width of 2.0 m and carriers a discharge of 4.80 m³/s with a depth of 1.60 m. at a certain section a small, smooth hump with a flat top and of height 0.10 m is proposed to be built. Calculate the likely change in the water surface. Neglect the energy loss.*

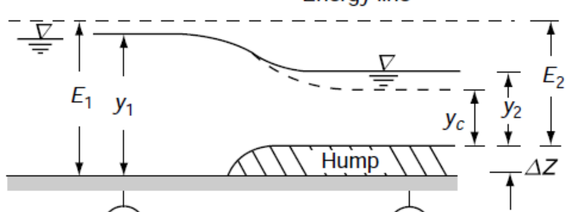


Fig. 2.9 *Channel transition with a hump*

$$q = \frac{4.80}{2.0} = 2.40 \text{ m}^3\text{/s/m} \qquad F_1 = V_1 / \sqrt{g y_1} = 0.379$$

$$V_1 = \frac{2.40}{1.6} = 1.50 \text{ m/s}, \quad \frac{V_1^2}{2g} = 0.115 \text{ m}$$

hence the upstream flow is subcritical

the hump will cause a drop in the watersurface elevation.

$$E_1 = 1.60 + 0.115 = 1.715 \text{ m}$$

At Section 2, $E_2 = E_1 - \Delta Z = 1.715 - 0.10 = 1.615$ m $\quad y_c = \left(\frac{(2.4)^2}{9.81}\right)^{1/3} = 0.837$ m $\quad E_c = 1.5\, y_c = 1.256$ m

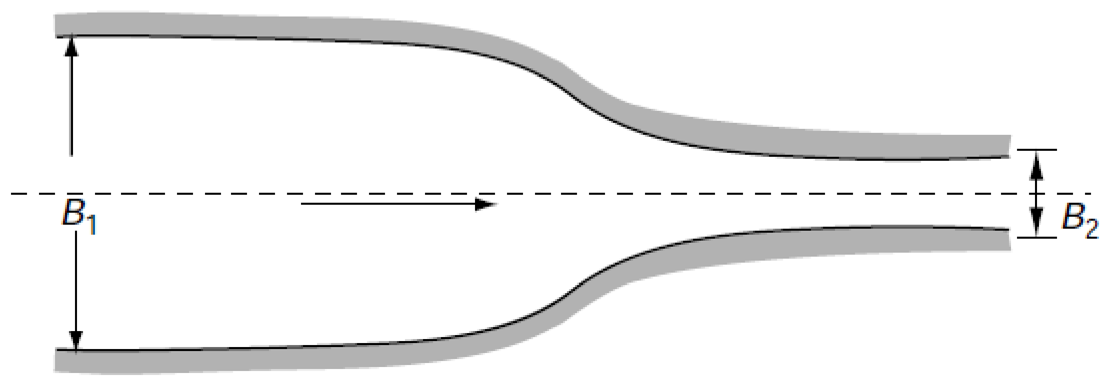
$$y_2 + \frac{V_2^2}{2g} = E_2 \qquad y_2 + \frac{(2.4)^2}{2 \times 9.81 \times y_2^2} = 1.615$$

Solving by trial-and-error, $y_2 = 1.481$ m.

TRANSITIONS WITH CHANGE IN WIDTH

(a) Subcritical flow in a Width Constriction

$$E_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{Q^2}{2gB_1^2 y_1^2}$$



$$E_2 = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{Q^2}{2gB_2^2 y_2^2}$$

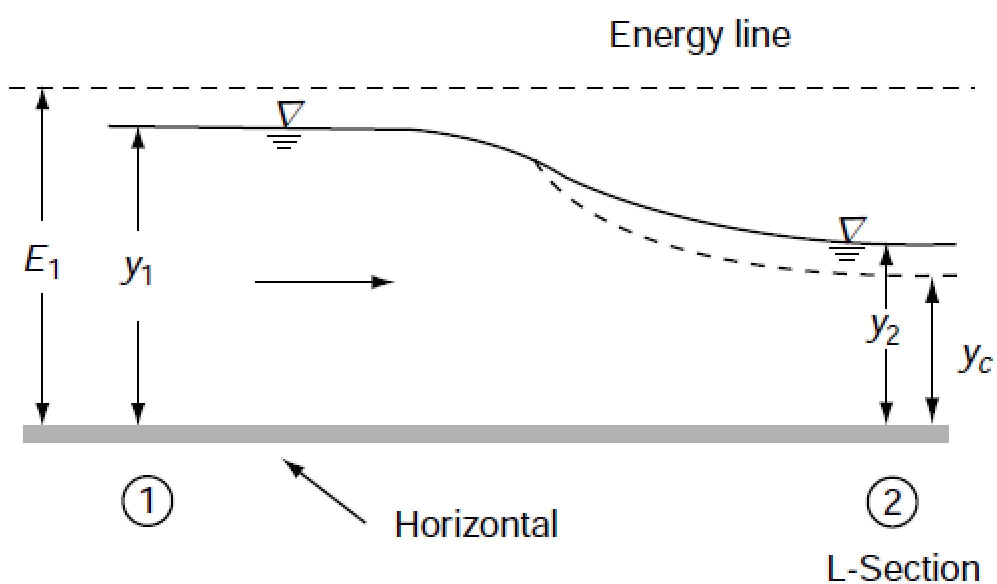


Fig. 2.14 *Transition with width constriction*

TRANSITIONS WITH CHANGE IN WIDTH

(a) Subcritical flow in a Width Constriction

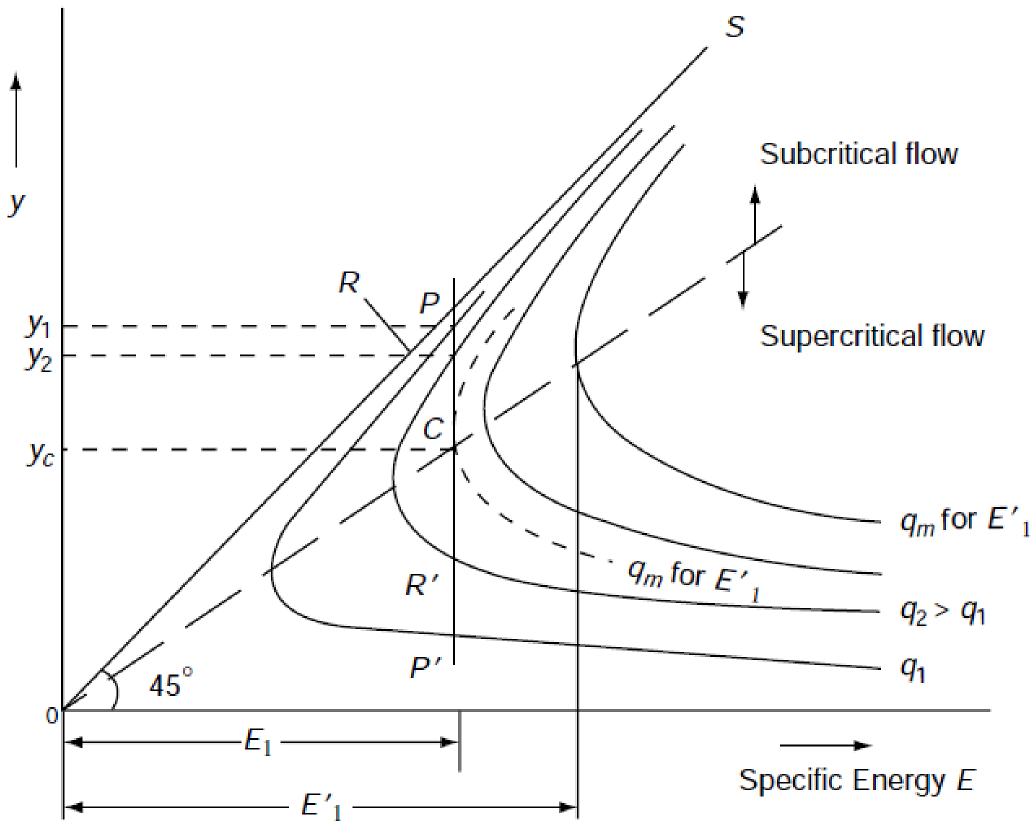


Fig. 2.15 *Specific energy diagram for transition of Fig. 2.14*

$$E_1 = E_{cm} = y_{cm} + \frac{Q^2}{2g(B_{2m})^2 y_{cm}^2}$$

For a rectangular channel, at critical flow

$$y_c = \frac{2}{3} E_c \qquad \text{Since } E_1 = E_{cm}$$

and $$y_c = \left(\frac{Q^2}{B_{2m}^2 g} \right)^{1/3}$$

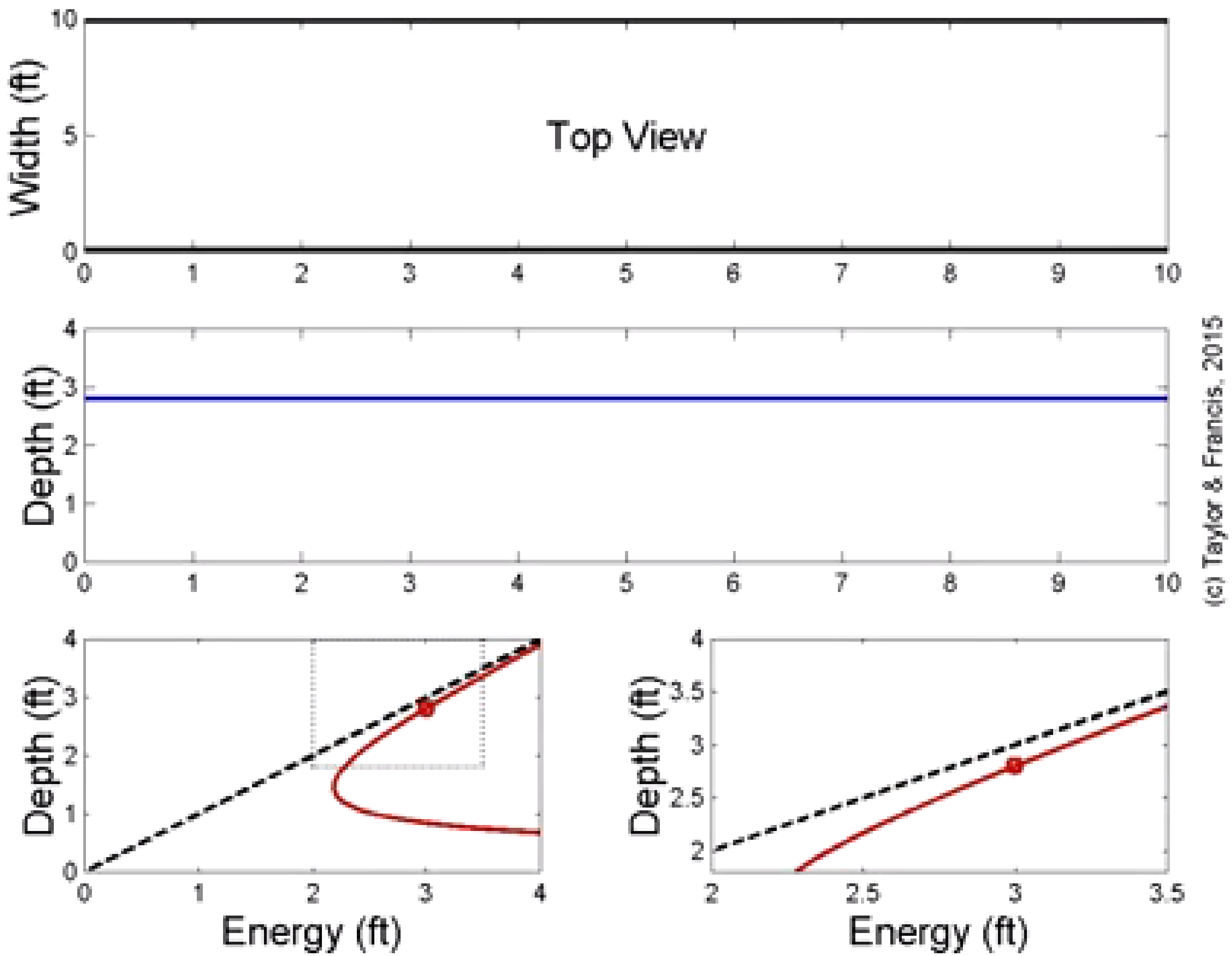
or $$B_{2m} = \sqrt{\frac{Q^2}{g\, y_{cm}^3}}$$

$$B_{2m} = \sqrt{\frac{27 Q^2}{8 g E_1^3}}$$

$$E_1' = y_1' + \frac{Q^2}{2g(B_1^2 y_1'^2)}$$

TRANSITIONS WITH CHANGE IN WIDTH

(a) Subcritical flow in a Width Constriction



TRANSITIONS WITH CHANGE IN WIDTH

(a) Subcritical flow in a Width Constriction

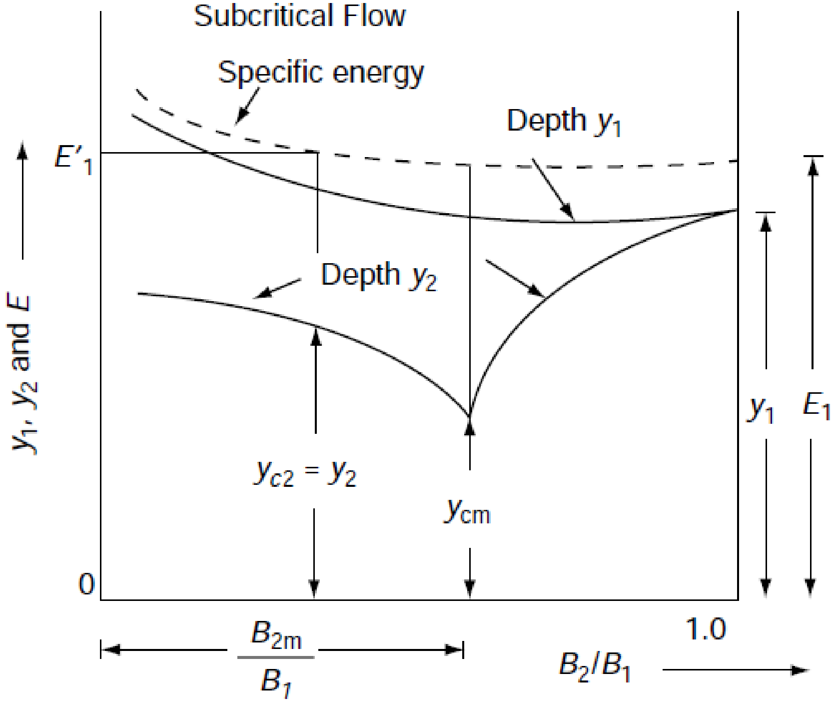


Fig. 2.16 *Variation of y_1 and y_2 in subcritical flow in a width constriction*

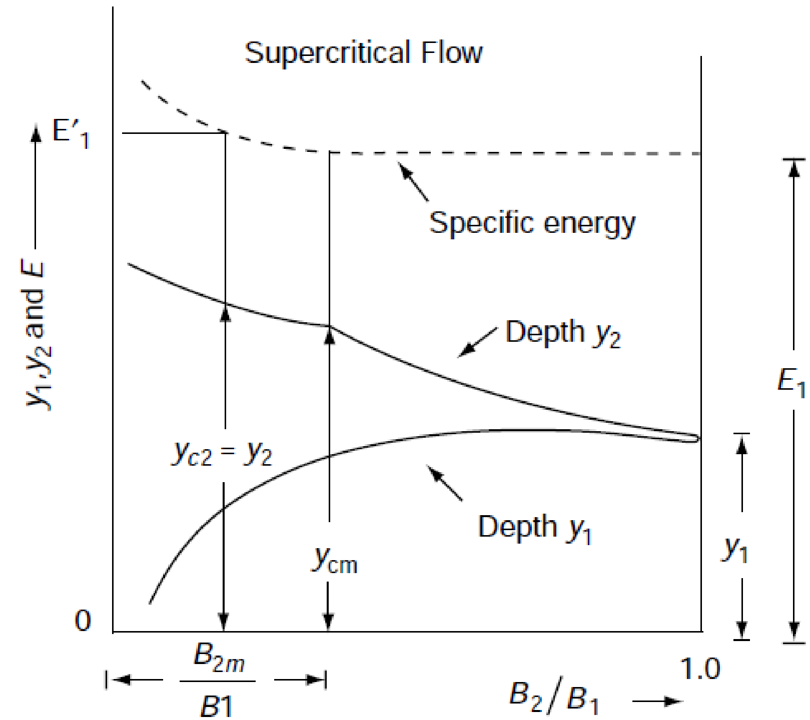


Fig. 2.17 *Variation of y_1 and y_2 in supercritical flow in a width constriction*

TODAY’S DEAL

ASSIGNMENT -3

SOLVE ANY 10 UNSOLVED QUESTIONS FROM THE CHAPTER : ENERGY DEPTH RELATIONSHIP

LAST DATE: 11TH OCTOBER 2020, SUNDAY

MAIL TO : hhmc2021@gmail.com

THE END